

Firing Costs and Business Cycle Fluctuations*

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Abstract: This paper considers a real business cycle model with establishment level dynamics and uses it to analyze the effects of firing taxes. It finds that the firing taxes have significant consequences on business cycle fluctuations. The largest effects are on aggregate employment, which becomes less variable and more persistent. While the business cycle effects of firing taxes are sizable, their welfare consequences are completely dominated by their steady state effects.

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1. Introduction

There is a large literature evaluating the consequences of firing costs on long-run labor market outcomes. Its main purpose: To explain the differences in unemployment rates observed between Europe and the U.S. While this an important objective and there has been considerable progress in this line of research, this paper changes the focus of the analysis and evaluates the effects of firing costs on business cycle dynamics. There are two reasons for doing this. First, policy makers often justify introducing firing costs as an effective tool for reducing the magnitude of economic downturns. Thus, it seems important to evaluate the rationale behind this claim. Second, there are features of the data, besides long-run labor market outcomes, that firing costs might help explain. In fact, employment protection levels do seem to be related to business cycle differences across countries: Figure 1 plots the standard deviation of output against the employment protection level for seventeen OECD countries, showing that they are negatively related. While this relation is highly suggestive, it does not provide evidence that firing costs affect business cycle dynamics in any important way. A third factor (e.g. high risk aversion) could be generating lower output fluctuations and leading countries to adopt higher employment protection levels. Thus, a negative relation in Figure 1 could be obtained while firing costs have no effects. To determine the effects of firing costs on business cycle fluctuations, analysis is needed. This paper provides such analysis: It describes a real business cycle model with establishment level dynamics, introduces firing costs and evaluates their effects.

The model is a stochastic version of Veracierto [17], which in turn is based on Hopenhayn and Rogerson [9]. The economy is populated by a representative household that values consumption and leisure. Output, which can be consumed or invested, is produced by a large number of establishments that use capital and labor as inputs into a decreasing returns to scale technology. Establishments are subject both to idiosyncratic and aggregate productivity shocks. In the benchmark case, both capital and labor are freely movable across establishments.

Once the benchmark model is parametrized to U.S. data, firing taxes ranging from one month to one year of wages are introduced. The paper finds that firing taxes have consid-

erable business cycle effects. In particular, firing taxes equal to one year of wages reduce the standard deviation of output by 10.7%, the standard deviation of investment by 14.7% and the standard deviation of employment by 30.6%. Also, the firing taxes make aggregate employment more persistent: its first order autocorrelation increases from 0.66 to 0.71. These findings suggest that firing costs could play a significant role in explaining differences in business cycle fluctuations across countries.

This is not the first paper analyzing the effects of firing costs on business cycle dynamics. In a partial equilibrium setting, Campbell and Fisher [4] studied how firing costs affect the aggregate behavior of a large number of establishments subject to idiosyncratic productivity shocks and a shock to the aggregate wage rate. Their focus was on the volatility of job destruction relative to the volatility of job creation, finding that firing costs increase it. Cabrales and Hopenhayn [3] analyzed a similar type of model except that the aggregate shock was in the aggregate productivity level instead of the wage rate. Contrary to Campbell and Fisher [4], they found that the firing costs decrease the volatility of job destruction relative to the volatility of job creation. This paper differs from Campbell and Fisher [4] and Cabrales and Hopenhayn [3] in that it performs a general equilibrium analysis where both aggregate productivity and the wage rate are changing. In terms of the relative volatility of job destruction, this paper obtains results that are closer to Cabrales and Hopenhayn [3] than to Campbell and Fisher [4].

Current work by Samaniego [15] is more closely related. Samaniego also considers a version of Veracierto [17] and performs a general equilibrium analysis.¹ However, he studies how firing taxes affect the deterministic transitional dynamics after a large persistent change in aggregate productivity. This paper, on the contrary, computes the full stochastic equilibrium of a real business cycle model. An advantage of this approach is that it allows to evaluate how firing taxes affect standard business cycle statistics. Another advantage is that it allows to assess the welfare benefits of reducing business cycle fluctuations. Despite the differences, all these papers share a basic result: Firing taxes are found to lower the response

¹The models differ in that Samaniego [15] allows for endogenous entry and exit while this paper treats them as exogenously determined. Another difference is that this paper gives firms a “quits allowance” before being subject to firing taxes, while Samaniego doesn’t.

of the economy to aggregate productivity changes.

The rest of the paper is organized as follows. Section 2 describes the environment. Section 3 defines a competitive equilibrium and describes the computational strategy. Section 4 calibrates the benchmark economy. Finally, Section 5 introduces firing taxes and evaluates their effects. An appendix provides the proofs to all the claims made in the paper.

2. The economy

The economy is populated by a unit measure of ex-ante identical agents with preferences given by

$$E \sum_{t=0}^{\infty} \beta^t [\ln c_t + v(l_t)],$$

where c_t is consumption, l_t is leisure and $0 < \beta < 1$ is the discount factor. Every period agents are endowed with ω units of time. Given an institutionally determined workweek of length equal to one, leisure can only take values ω or $\omega - 1$.²

Output, which can be consumed or invested, is produced by a large number of establishments with production function given by

$$y_t = e^{z_t} s_t g_t^\theta n_t^\gamma,$$

where $z_t \in Z$ is an aggregate productivity shock, $s_t \in S = \{0, s_{\min}, \dots, s_{\max}\}$ is an idiosyncratic productivity shock, g_t is capital, n_t is labor, $\theta > 0$, $\gamma > 0$, and $\theta + \gamma < 1$. The aggregate productivity shock $z_t \in Z$, which is common to all establishments, follows a finite Markov process with transition matrix H . The idiosyncratic productivity shock $s_t \in S$ also follows a finite Markov process, but with transition matrix Q . Realizations of s_t are assumed to be independent across all establishments and $s_t = 0$ is assumed to be an absorbing state. Since there are no fixed costs of operation, establishments will exit only when their idiosyncratic productivity becomes zero. Every period ν new establishments are exogenously born. The distribution over initial idiosyncratic productivity levels is given by ψ .

²In order to analyze the effects of firing taxes it is important to assume that labor is indivisible: It allows to associate changes in the labor input of establishments with changes in employment.

3. Competitive equilibrium

In this section I describe a competitive equilibrium where establishments are subject to firing taxes and the proceeds are rebated to households as lump sum transfers. Following Hopenhayn and Rogerson [9], firing taxes are modelled as a tax on reducing employment. In particular, whenever an establishment makes its current employment level n_t lower than $(1 - q)n_{t-1}$ it must pay a tax rate τ on the difference. Observe that q is a policy parameter specifying a contraction rate below which establishments are not subject to firing taxes. Hereon, I will refer to q as the “quit rate of workers”.³

In order to define a competitive equilibrium I will index the history of an individual establishment by $s^a = (s_0, \dots, s_a) \in S^{a+1}$, where s_j is the idiosyncratic productivity that the establishment had when it was of age j . Also, the history of aggregate productivity levels since date 0 will be denoted by $z^t = (z_0, \dots, z_t) \in Z^{t+1}$, where z_j is the aggregate productivity level that the economy had at date j .

Following Hansen [7] and Rogerson [14], I assume that agents trade employment lotteries. This makes the preferences of the representative household linear with respect to the probability of working η_t .⁴ The problem of the representative household at date 0 is then given by the following equation:

$$\max \left\{ \ln c_0 - \alpha \eta_0 + \sum_{t=1}^{\infty} \sum_{z^t} \beta^t [\ln c_t(z^t) - \alpha \eta_t(z^t)] \left[\prod_{j=1}^t H(z_{j-1}, z_j) \right] \right\} \quad (3.1)$$

subject to:

$$\begin{aligned} c_t(z^t) + k_{t+1}(z^t) - (1 - \delta) k_t(z^{t-1}) + \sum_{z_{t+1}} p_t(z^t; z_{t+1}) b_{t+1}(z^t, z_{t+1}) \\ \leq w_t(z^t) \eta_t(z^t) + r_t(z^t) k_t(z^{t-1}) + b_t(z^t) + D_t(z^t) + T_t(z^t) \end{aligned} \quad (3.2)$$

³The parameter q will be actually calibrated to the quit rate of workers since, in practice, establishments do not have to pay firing taxes on quits. Assuming a positive q is not only a considerable gain in realism, but will make the problem of computing a competitive equilibrium tractable.

⁴In particular, α in equation (3.1) is given by $v(\omega) - v(\omega - 1)$.

$b_0 = 0$, k_0 , and z_0 given,

where k_t is the capital owned by the household, $p_t(\cdot, z_{t+1})$ is the price of an Arrow security which delivers one unit of the consumption good if z_{t+1} is realized, $b_t(\cdot, z_{t+1})$ are the purchases of this type of security, w_t is the wage rate, r_t is the rental rate of capital, D_t are profits and T_t are the lump sum transfers from the government.

Establishments maximize expected discounted profits net of firing taxes. The problem of an establishment of age a and idiosyncratic history s^a (when the aggregate history is given by z^t) is described by the following equation:

$$\begin{aligned} & \max \{ e^{z^t} s_a g_{a,t} (s^a, z^t)^\theta n_{a,t} (s^a, z^t)^\gamma - w_t (z^t) n_{a,t} (s^a, z^t) - r_t (z^t) g_{a,t} (s^a, z^t) - \tau f_{a,t} (s^a, z^t) \\ & + \sum_{j=1}^{\infty} \sum_{s^{a+j}} \sum_{z^{t+j}} \left[\prod_{h=1}^j p_{t+h-1} (z^{t+h-1}; z_{t+h}) \right] [e^{z^{t+j}} s_{a+j} g_{a+j,t+j} (s^{a+j}, z^{t+j})^\theta n_{a+j,t+j} (s^{a+j}, z^{t+j})^\gamma \\ & - w_{t+j} (z^{t+j}) n_{a+j,t+j} (s^{a+j}, z^{t+j}) - r_{t+j} (z^{t+j}) g_{a+j,t+j} (s^{a+j}, z^{t+j}) \\ & - \tau f_{a+j,t+j} (s^{a+j}, z^{t+j})] \left[\prod_{h=1}^j Q(s_{a+h-1}, s_{a+h}) \right] \} \end{aligned}$$

subject to:

$$n_{a+j,t+j} (s^{a+j}, z^{t+j}) \geq (1 - q) n_{a+j-1,t+j-1} (s^{a+j-1}, z^{t+j-1}) - f_{a+j,t+j} (s^{a+j}, z^{t+j}), \quad (3.3)$$

$$f_{a+j,t+j} (s^{a+j}, z^{t+j}) \geq 0, \quad (3.4)$$

$$n_{a-1,t-1} (s^{a-1}, z^{t-1}) \text{ given,}$$

where f is the amount of firing done by the establishment. Observe that the establishment cannot reduce its employment level below its previous period employment level (net of quits) without firing workers and paying the associated taxes. Although the above problem was defined for any initial condition, it must be the case that

$$n_{a-1,t-1} (s^{a-1}, z^{t-1}) = 0, \text{ when } a = 0, \quad (3.5)$$

since establishments are born with zero previous period employment. Also, observe that at $t = 0$, establishments of age a and history s^a take their previous employment level $n_{a-1,-1}(s^{a-1}, z^{-1})$ as given.

In order to aggregate the behavior of all establishments it will be important to describe the distribution μ of establishments across ages a and idiosyncratic histories s^a . This distribution satisfies the following equations:

$$\begin{aligned}\mu_{a+1}(s^{a+1}) &= Q(s_a, s_{a+1})\mu_a(s^a), \text{ for every } a \geq 0 \text{ and } s^{a+1}, \\ \mu_0(s^0) &= \nu\psi(s_0).\end{aligned}$$

Observe that the number of establishments of age 0 and productivity s_0 is given by the arrival of new establishments ν times the probability of drawing an initial productivity equal to s_0 .

The consumption good market clearing condition is then given by

$$c_t(z^t) + k_{t+1}(z^t) - (1 - \delta)k_t(z^{t-1}) = \sum_{a \geq 0} \sum_{s^a} e^{z^t s_a} g_{a,t}(s^a, z^t)^\theta n_{a,t}(s^a, z^t)^\gamma \mu_a(s^a). \quad (3.6)$$

This condition states that aggregate consumption plus aggregate investment must be equal to the production of all establishments.

The capital market clearing condition is

$$\sum_{a \geq 0} \sum_{s^a} g_{a,t}(s^a, z^t) \mu_a(s^a) = k_t(z^{t-1}). \quad (3.7)$$

That is, the total amount of capital rented by the establishments must be equal to the stock of capital supplied by the families.

Similarly, the market clearing condition for the labor market is given by

$$\sum_{a \geq 0} \sum_{s^a} n_{a,t}(s^a, z^t) \mu_a(s^a) = \eta_t(z^t). \quad (3.8)$$

The securities market clearing condition is simply

$$b_{t+1}(z^{t+1}) = 0, \quad (3.9)$$

since households are identical.

As was already mentioned, the government rebates to the households all the firing taxes collected from the establishments. The budget constraint of the government is then the following:

$$T_t(z^t) = \tau \sum_{a \geq 0} \sum_{s^a} f_{a,t}(s^a, z^t) \mu_a(s^a). \quad (3.10)$$

Finally, the profits received by the representative household must be equal to the profits made by all the establishments in the economy:

$$\begin{aligned} D_t(z^t) = \sum_{a \geq 0} \sum_{s^a} [e^{z^t} s_a g_{a,t}(s^a, z^t)^\theta n_{a,t}(s^a, z^t)^\gamma - w_t(z^t) n_{a,t}(s^a, z^t) \\ - r_t(z^t) g_{a,t}(s^a, z^t) - \tau f_{a,t}(s^a, z^t)] \mu_a(s^a). \end{aligned} \quad (3.11)$$

3.1. A quasi-planner equilibrium

While the competitive equilibrium with firing taxes described above seems a difficult object to analyze, it can be simplified quite substantially. It is straightforward to show that if $\{c_t, k_{t+1}, \eta_t, g_t, n_t, f_t, b_{t+1}, w_t, r_t, p_t, D_t, T_t\}_{t=0}^\infty$ is a competitive equilibrium, then $\{c_t, k_{t+1}, \eta_t, g_t, n_t, f_t\}_{t=0}^\infty$ solves the following quasi-planner problem:

$$\max \left\{ \ln c_0 - \alpha \eta_0 + \sum_{t=1}^{\infty} \sum_{z^t} \beta^t [\ln c_t(z^t) - \alpha \eta_t(z^t)] \left[\prod_{j=1}^t H(z_{j-1}, z_j) \right] \right\} \quad (3.12)$$

subject to

$$\begin{aligned} c_t(z^t) + k_{t+1}(z^t) - (1 - \delta) k_t(z^{t-1}) \\ \leq \sum_{a \geq 0} \sum_{s^a} \left[e^{z^t} s_a g_{a,t}(s^a, z^t)^\theta n_{a,t}(s^a, z^t)^\gamma - \tau f_{a,t}(s^a, z^t) \right] \mu_a(s^a) + T_t(z^t) \end{aligned} \quad (3.13)$$

$$n_{a,t}(s^a, z^t) \geq (1 - q) n_{a-1,t-1}(s^{a-1}, z^{t-1}) - f_{a,t}(s^a, z^t), \quad (3.14)$$

$$\sum_{a \geq 0} \sum_{s^a} n_{a,t}(s^a, z^t) \mu_a(s^a) = \eta_t(z^t) \quad (3.15)$$

$$\sum_{a \geq 0} \sum_{s^a} g_{a,t}(s^a, z^t) \mu_a(s^a) = k_t(z^{t-1}) \quad (3.16)$$

$$f_{a,t}(s^a, z^t) \geq 0, \quad (3.17)$$

$$n_{a-1,t-1}(s^{a-1}, z^{t-1}) = 0, \text{ for } a = 0 \quad (3.18)$$

k_0, z_0 , and $\{n_{a-1,-1}(s^{a-1}, z^{-1})\}_{a,s^a}$ given.

The converse is also true. If $\{c_t, k_{t+1}, \eta_t, g_t, n_t, f_t\}_{t=0}^\infty$ solves the above quasi-planner problem for some stochastic process T_t and the following condition is satisfied

$$T_t(z^t) = \tau \sum_{a \geq 0} \sum_{s^a} f_{a,t}(s^a, z^t) \mu_a(s^a), \quad (3.19)$$

then $\{c_t, k_{t+1}, \eta_t, g_t, n_t, f_t, b_{t+1}, w_t, r_t, p_t, D_t, T_t\}_{t=0}^\infty$ is a competitive equilibrium for some $\{b_{t+1}, w_t, r_t, p_t, D_t\}_{t=0}^\infty$.⁵

3.2. A recursive competitive equilibrium

In order to compute a competitive equilibrium it will be useful to work with a recursive formulation to the quasi-planner equilibrium described above. Since the quasi-planner problem (3.12) is convex, establishments that have different idiosyncratic histories and/or ages but that have identical previous period employment and current idiosyncratic productivity levels will be treated as being identical by the quasi-planner, i.e. they will be assigned the same contingent employment plan.⁶ As a result, in the recursive formulation that follows, I will index establishments by their previous period employment level u and their current idiosyncratic productivity level s .

The individual state of the representative quasi-planner is then given by the stock of capital k and a measure x describing the distribution of establishments across types (u, s) .

⁵Appendix A provides a formal proof for this equivalence result.

⁶For a proof, see Appendix B.

The aggregate state of the economy is given by the economy-wide capital level K , the economy-wide distribution of establishments X , and the aggregate productivity shock z . The problem faced by the representative quasi-planner is given by the following dynamic programming problem:

$$v(z, K, X, k, x) = \max_{c, \eta, g, n, i} \left\{ \ln c - \alpha \eta + \beta \sum_{z'} v(z', K', X', k', x') H(z, z') \right\}$$

subject to

$$c + i \leq \int \left\{ e^z s g(u, s)^\theta n(u, s)^\gamma - \tau \max[0, (1 - q)u - n(u, s)] \right\} dx + T(z, K, X) \quad (3.20)$$

$$\int n(u, s) dx \leq \eta \quad (3.21)$$

$$\int g(u, s) dx \leq k \quad (3.22)$$

$$k' = (1 - \delta)k + i \quad (3.23)$$

$$x'(U' \times \{s'\}) = \int_{(u, s): n(u, s) \in U'} Q(s, s') dx + \chi(0 \in U') \nu \psi(s') \quad (3.24)$$

$$T(z, K, X) = \int \tau \max[0, (1 - q)u - N(u, s; z, K, X)] dX \quad (3.25)$$

$$K' = (1 - \delta)K + I(z, K, X) \quad (3.26)$$

$$X'(U' \times \{s'\}) = \int_{(u, s): N(u, s; z, K, X) \in U'} Q(s, s') dx + \chi(0 \in U') \nu \psi(s') \quad (3.27)$$

where χ is an indicator function that takes value equal to one if the argument is true and zero otherwise. Observe that, aside from the aggregate state of the economy (z, K, X) , the representative quasi-planner takes the economy-wide employment decision rule N and economy-wide investment decision rule I as given.

In a recursive competitive equilibrium, expectations must be rational:

$$N(u, s; z, K, X) = n(u, s; z, K, X, K, X)$$

and

$$I(z, K, X) = i(z, K, X, K, X).$$

That is, the economy-wide decision rules N and I must be generated by the decision rules n and i of the representative quasi-planner.

3.3. Computational strategy

Observe that, conditional on n , k , and z , the optimal capital allocation rule g is obtained by maximizing aggregate output $\int e^z s g^\theta n^\gamma dx$ subject to the feasibility constraint (3.22). Substituting this solution and equation (3.25) into equation (3.20) and then substituting the resulting expression together with equation (3.21) into the one-period return function

$$R = \ln c_t - \alpha \eta_t$$

allows to write the return function as a function of (z, K, X, k, x, N, n, i) .

The problem of the representative quasi-planner can then be written as

$$v(z, K, X, k, x) = \max_{n, i} \{R(z, K, X, k, x, N, n, i) + \beta E[v(z', K', X', k', x') | z]\} \quad (3.28)$$

subject to equations (3.23), (3.24), (3.26) and (3.27).

The high dimensionality of the state space seems to preclude any possibility of computing a recursive competitive equilibrium. However, two features of the problem will render it tractable. The first is the nature of the employment decision rule n . Appendix C shows that the employment decision rule is fully characterized by a pair of threshold functions \bar{n} and \underline{n} as follows

$$n(u, s; z, K, X, k, x) = \left\{ \begin{array}{l} \bar{n}(s; z, K, X, k, x), \text{ if } (1 - q)u > \bar{n}(s; z, K, X, k, x) \\ \underline{n}(s; z, K, X, k, x), \text{ if } (1 - q)u < \underline{n}(s; z, K, X, k, x) \\ (1 - q)u, \text{ otherwise} \end{array} \right\}. \quad (3.29)$$

Observe that the upper and lower thresholds \bar{n} and \underline{n} do not depend on the previous employment level u . The (S,s) nature of the employment decision rule is critical for making the

decision variables in (3.28) finite dimensional: instead of letting the quasi-planner choose a generic function n , there will be no loss of generality in constraining it to choose finite dimensional thresholds \bar{n} and \underline{n} and defining the employment decision rule n as in equation (3.29).

The second property that makes the problem tractable is that, if the aggregate productivity z fluctuations are sufficiently small, along a stationary equilibrium the distribution x will always have a finite support. To see this more clearly it will be convenient to consider the deterministic steady state of an economy where the aggregate productivity level z is constant and equal to zero. Hereon, any variable superscripted with a star (*) will refer to its corresponding deterministic steady state value. Before proceeding I state the following result.

Proposition 3.1. *In a deterministic steady state equilibrium, the invariant distribution x^* has a finite support given by the union of $\{0\}$ and the following set:*

$$m^* = \left\{ (1-q)^h \underline{n}^*(s) \right\}_{\substack{s=s_{\min}, \dots, s_{\max} \\ h=1, \dots, \underline{\Omega}(s)}} \cup \left\{ (1-q)^h \bar{n}^*(s) \right\}_{\substack{s=s_{\min}, \dots, s_{\max} \\ h=1, \dots, \bar{\Omega}(s)}}$$

where $\underline{\Omega}(s)$ is the lowest natural number satisfying that

$$(1-q)^{\underline{\Omega}(s)} \underline{n}^*(s) < \underline{n}^*(s_{\min})$$

and $\bar{\Omega}(s)$ is the lowest natural number satisfying that

$$(1-q)^{\bar{\Omega}(s)} \bar{n}^*(s) < \underline{n}^*(s_{\min})$$

Proof: See Appendix D.

Hereon, I will assume that m^* is a vector conveniently ordered. I will refer to $m^*(j)$ as the j th element of m^* and the total number of elements in m^* will be denoted by J . Also, it will be useful to classify the elements of m^* into three sets: 1) those that correspond to establishments that expand (set \mathcal{G}^*), 2) those that correspond to establishments that contract (set \mathcal{C}^*), and 3) those that correspond to establishments that remain inactive (set

\mathcal{I}^*). That is, for $j = 1, \dots, J$:

$$\begin{aligned}
j &\in \mathcal{G}^*, \text{ if } m^*(j) = (1 - q)\underline{n}^*(s) \text{ for some } s \geq s_{\min} \\
j &\in \mathcal{C}^*, \text{ if } m^*(j) = (1 - q)\bar{n}^*(s) \text{ for some } s \geq s_{\min} \\
j &\in \mathcal{I}^*, \text{ if } m^*(j) = (1 - q)m(j - 1)
\end{aligned} \tag{3.30}$$

Observe that equation (3.30) defines an implicit ordering for m^* .

Suppose that, at some date t , the state variable x_t has a finite support given by m_t (and the singleton $\{0\}$), that m_t has dimension J (same dimension as m^*), that m_t is close to m^* and that

$$x_t(\{0\}, s) = x^*(\{0\}, s), \text{ for every } s \in S \tag{3.31}$$

$$x_t(\{m_t(j)\}, s) = x^*(\{m^*(j)\}, s), \text{ for every } s \in S \text{ and every } j = 1, \dots, J \tag{3.32}$$

$$x_t = 0, \text{ everywhere else.}$$

In addition, assume that \underline{n}_t and \bar{n}_t are close to their steady state values \underline{n}^* and \bar{n}^* .⁷ Then, the next period finite support m_{t+1} will be given by

$$m_{t+1}(j) = \left\{ \begin{array}{l} (1 - q)\underline{n}_t(s), \text{ if } j \in \mathcal{G}^* \\ (1 - q)\bar{n}_t(s), \text{ if } j \in \mathcal{C}^* \\ (1 - q)m_t(j - 1), \text{ if } j \in \mathcal{I}^* \end{array} \right\}, \text{ for } j = 1, \dots, J, \tag{3.33}$$

where s in the first line satisfies that $m^*(j) = (1 - q)\underline{n}^*(s)$ and s in the second line satisfies that $m^*(j) = (1 - q)\bar{n}^*(s)$. By continuity, m_{t+1} will be close to m^* and x_{t+1} will satisfy that

$$x_{t+1}(\{0\}, s) = x^*(\{0\}, s), \text{ for every } s \in S$$

$$x_{t+1}(\{m_{t+1}(j)\}, s) = x^*(\{m^*(j)\}, s), \text{ for every } s \in S \text{ and every } j = 1, \dots, J$$

$$x_{t+1} = 0, \text{ everywhere else.}$$

⁷For $s = 0$ I assume without loss of generality that $\underline{n}(s) = \bar{n}(s) = 0$, i.e. that the employment thresholds are identical to their deterministic steady state values.

Assuming that $\underline{n}_t, \bar{n}_t, m_t, k_t, i_t, \underline{N}_t, \bar{N}_t, M_t, K_t, I_t$ fluctuate in a small neighborhood of their deterministic steady state values, the original representative quasi-planner problem (3.28) can then be replaced by the following transformed problem:

$$v(z, K, M, k, m) = \max_{\underline{n}, \bar{n}, i} \left\{ \tilde{R}(z, K, M, k, m, \underline{N}, \bar{N}, I, \underline{n}, \bar{n}, i) + \beta E[v(z', K', M', k', m') \mid z] \right\} \quad (3.34)$$

subject to

$$m'(j) = \begin{cases} (1-q)\underline{n}(s), & \text{if } j \in \mathcal{G}^* \\ (1-q)\bar{n}(s), & \text{if } j \in \mathcal{C}^* \\ (1-q)m(j-1), & \text{if } j \in \mathcal{I}^* \end{cases}, \text{ for } j = 1, \dots, J,$$

$$k' = (1-\delta)k + i$$

$$M'(j) = \begin{cases} (1-q)\underline{N}(s; z, K, M), & \text{if } j \in \mathcal{G}^* \\ (1-q)\bar{N}(s; z, K, M), & \text{if } j \in \mathcal{C}^* \\ (1-q)M(j-1), & \text{if } j \in \mathcal{I}^* \end{cases}, \text{ for } j = 1, \dots, J,$$

$$K' = (1-\delta)K + I(z, K, M).$$

where the decision variables \underline{n} and \bar{n} are defined over $s \geq s_{\min}$.⁸ The conditions for a recursive competitive equilibrium now become:

$$\underline{N}(s; z, K, M) = \underline{n}(s; z, K, M, K, M)$$

$$\bar{N}(s; z, K, M) = \bar{n}(s; z, K, M, K, M)$$

$$I(z, K, M) = i(z, K, M, K, M).$$

The return function \tilde{R} in (3.34) is given by the value of the return function R in (3.28) that corresponds to the following variables: 1) the discrete distribution x is defined by the finite support m as in equation (3.31), 2) the employment rule n is defined by the employment thresholds \underline{n} and \bar{n} as in (3.29), 3) the discrete distribution X is defined by the finite support

⁸Without loss of generality, $\underline{n}(0)$ and $\bar{n}(0)$ are set identical to zero. If τ is sufficiently small relative to the present discounted value of wages, this will always be the optimal choice.

M , and 3) the employment rule N is defined by the employment thresholds \underline{N} and \bar{N} . The advantage of working with the transformed problem (3.34) instead of the original problem (3.28) is that it has linear laws of motion. Since all the endogenous arguments of \tilde{R} take strictly positive values in the deterministic steady state, a second order Taylor expansion around the deterministic steady state can be performed to obtain a quadratic return function. This delivers a linear-quadratic recursive competitive equilibrium structure that can be solved using standard techniques (e.g. Hansen and Prescott [8]). The assumption that $\underline{n}_t, \bar{n}_t, m_t, k_t, i_t, \underline{N}_t, \bar{N}_t, M_t, K_t, I_t$ fluctuate in a sufficiently small neighborhood of their deterministic steady state values is satisfied in all the experiments reported in this paper.

4. Parametrization

This section describes the steady state observations used to calibrate the model parameters.⁹ Since the model will be calibrated to U.S. data and this economy is characterized by low firing costs, the parameter τ is set to zero.¹⁰ Given τ , the rest of the parameters to be calibrated are $\beta, \theta, \gamma, \nu, q, \alpha, \delta$, the distribution ψ , the transition matrix Q for the idiosyncratic productivity shocks, and the transition matrix H for the aggregate productivity shock. The model time period is selected to be one quarter.

The first issue that must be addressed is what actual measure of capital should the model capital correspond to. Since the focus is on establishment level dynamics, it seems natural to abstract from capital components such as land, residential structures, and consumer durables. The empirical counterpart for capital is then identified with plant, equipment, and inventories. As a result, investment is associated in the NIPA with nonresidential investment plus changes in business inventories. The empirical counterpart for consumption is identified with personal consumption expenditures in nondurable goods and services. Output is then defined as the sum of these investment and consumption measures. The quarterly capital-output ratio and the investment-output ratio corresponding to these measures are 6.8 and 0.15, respectively. Since, at steady state $I/Y = \delta(K/Y)$, these ratios require that $\delta = 0.0221$.

⁹The calibration procedure follows Veracierto [17] quite closely.

¹⁰In the next section, the firing cost parameter τ will be increased and its effects analyzed.

The annual interest rate is selected to be 4 per cent, which is a compromise between the average real return on equity and the average real return on short-term debt for the period 1889 to 1978 as reported by Mehra and Prescott [11]. The discount factor β is then chosen to be 0.99 in order to generate this annual interest rate.

Given the above values for β and δ , and given that the capital share satisfies

$$\theta = \frac{(1/\beta + \delta) K}{Y},$$

matching the U.S. capital-output ratio requires choosing a value of θ equal to 0.2186. Similarly, $\gamma = 0.64$ is selected to generate the share of labor in the National Income and Product Accounts.

The disutility of work parameter α is an important determinant of aggregate employment η . Thus, $\alpha = 0.94$ is picked so that 80 percent of the population works at steady state, roughly the fraction of the U.S. working age population that is employed.

In turn, the quarterly quit rate parameter q is chosen to be 6 per cent, which is consistent with evidence on quits from the Job Openings and Labor Turnover Survey (JOLTS) published by the Bureau of Labor Statistics.

The transition matrix for the idiosyncratic productivity levels Q is restricted to be a finite approximation to a continuous process of the following form:

$$\Pi(0, \{0\}) = 1$$

$$\Pi(s, [s_{\min}, \hat{s}]) = \frac{1}{\zeta} \Pr \{ (a + \rho_s \ln s + \varepsilon'_s) \in [s_{\min}, \hat{s}] \}, \text{ for } s, \hat{s} \geq s_{\min}$$

where a , ρ_s and ζ are constants, ε'_s is an i.i.d. normally distributed variable with mean 0 and standard deviation σ_s , and $\Pi(s, A)$ is the probability of transiting from s to a next period value in the set A .¹¹ We then have to determine the four parameters a , ρ_s , ζ and σ_s , the idiosyncratic productivity levels $\{s_{\min}, \dots, s_{\max}\}$ and the initial distribution ψ . Since all these parameters are important determinants of the establishment dynamics of the model, their

¹¹Observe that Π is basically an AR(1) process truncated at the value of 0.

values will be selected to reproduce several features of U.S. establishment dynamics.

One such feature is the distribution of establishments by employment size as reported by the Census of Manufacturers. In particular, the distribution over initial idiosyncratic productivity levels ψ is selected so that the invariant distribution x^* in the model economy mimics the average size distribution of manufacturing establishments across the census years 1967, 1972, 1977 and 1982, which is reproduced in Table 1. For this purpose, a total of nine positive idiosyncratic productivity levels are introduced and their values $\{s_{\min}, \dots, s_{\max}\}$ are selected so that the (corresponding nine types of) establishments in the model economy display employment levels in the middle of each of the employment ranges shown in Table 1.¹²

Another set of observations on (manufacturing) establishment dynamics pertains to job-creation and job-destruction data. Davis and Haltiwanger [5] reported that, for the period between 1972:2 and 1988:4, the job-creation rate due to births (JCB) was 0.62% while the job-creation rate due to continuing establishments (JCC) was 4.77%. They also reported that the job-destruction rate due to deaths (JDD) was 0.83% while the job-destruction rate due to continuing establishments (JDC) was 4.89%.¹³ Since employment is stationary in the model economy, the model can not match these exact job-creation and job-destruction rates. Imposing the approximate symmetry observed in U.S. data, I chose instead to match the following rates: JCB = 0.73, JCC = 4.80%, JDD = 0.73% and JDC = 4.80%. This gives rise to three independent observations. In order to calibrate the four parameters a , ρ_s , ζ and σ_s associated to the transition matrix an additional observation is then needed.

The last observation is obtained from Dunne et al. [6] who analyzed establishment turnover using data on plants that first began operating in the 1967, 1972, and 1977 Census of Manufacturing. They found that the five-year exit rate among these establishments was 36.2%. Matching this exit rate, together with the job-creation and destruction rates described above, requires the following parameter values: $a = 0.05155$, $\rho_s = 0.996$, $\zeta = 1.005$ and $\sigma_s = 0.0372$. The values for the idiosyncratic productivity levels $\{s_{\min}, \dots, s_{\max}\}$, the initial

¹²In practice, I normalized the lowest idiosyncratic productivity level s_{\min} to one and chose the endowment of new establishments ν to make the nine employment levels fall in the middle of the employment ranges.

¹³These are all quarterly rates.

distribution ψ and the transition matrix Q that correspond to this calibration procedure are provided in Table 2.

Finally, the aggregate productivity shock is constrained to follow a standard AR(1) process:

$$z' = \rho_z z + \varepsilon'_z$$

where ε'_z is an i.i.d. normally distributed variable with mean 0 and standard deviation σ_z .¹⁴ The parameters ρ_z and σ_z are selected so that measured Solow residuals in the model economy replicate the behavior of measured Solow residuals in the data.¹⁵ Using the measure of output described above and a labor share of 0.64, measured Solow residuals are found to be as highly persistent as in Prescott [13] but the standard deviation of technology changes is somewhat smaller: 0.0063 instead of the usual 0.0076 value used in the literature. As a consequence, $\rho_z = 0.95$ and $\sigma_z = 0.0063$ are chosen here.

5. Results

5.1. Steady state effects

This section reports the steady state effects of firing taxes in the deterministic version of the model economy.¹⁶ Providing a steady state analysis is important because it describes how the firing taxes affect the mean levels around which the economy fluctuates.

Table 3 shows the effects of increasing the firing tax τ from zero to 0.33, one, two and four quarters of wages.¹⁷ We see that the steady state consequences on the job reallocation

¹⁴Instead of selecting a finite approximation to this process (which would determine the finite set Z and the transition matrix H described in the previous sections) I choose to work with the continuous AR(1) process directly since the linear-quadratic computational method renders it tractable.

¹⁵Proportionate changes in measured Solow residual are defined as the proportionate change in aggregate output minus the sum of the proportionate change in labor times the labor share γ , minus the sum of the proportionate change in capital times $(1 - \gamma)$.

¹⁶This type of analysis is not novel. A number of papers have evaluated the steady state effects of firing taxes in a variety of settings. Alvarez and Veracierto [2], Hopenhayn and Rogerson [9], Millard and Mortensen [12] and Veracierto [17] are only a few examples.

¹⁷Firing costs equal to one year of wages amount to the severance payments that must be given to blue collar workers with ten years of service in countries with the toughest legislation (Lazear [10]). That is, they

process are quite significant. In order to avoid paying firing taxes equal to one year of wages ($\tau = 4w$), the job destruction rate of continuing establishments (JDC) decreases from 4.80% to 2.74%. Since establishments prefer to wait until they exit before firing additional workers, the job destruction rate due to deaths (JDD) increases from 0.73% to 0.80%. In turn, establishments that receive positive productivity shocks choose to reduce their employment growth in order to avoid paying firing taxes in the future. This leads to a reduction in the job creation rate due to continuing establishments (JCC) from 4.80% to 3.05% and in the job creation rate due to births (JCB) from 0.73% to 0.47%. The fact that establishments do not respond to the idiosyncratic productivity shocks as much as they do in the absence of firing taxes leads to a significant production inefficiency: Establishments with low productivity levels end up employing too many workers and establishments with high productivity levels end up employing too few workers. This production inefficiency induces agents to substitute away from market activities towards leisure, leading to a decrease of 2.46% in aggregate employment. The lower productivity and employment levels in turn lead to a decrease of 3.52% in output, consumption, capital and investment.

The last row of Table 3 shows the welfare effects. In particular, it reports the proportionate increase in consumption that must be given to the representative agent living in the steady state with firing taxes to make him indifferent with being in the steady state with no firing taxes. Since the economy without firing taxes is Pareto optimal, we know that this compensation must be positive. In fact, Table 3 shows that it can be a large number: According to this measure, the welfare cost of introducing a firing tax equal to one year of wages is equal to 1.74% of consumption.¹⁸

5.2. Business cycle effects

This section constitutes the core of the paper. It analyzes the effects of firing taxes on business cycle dynamics. Before proceeding to the main results, it will be important to determine

represent an upper bound on what is empirically reasonable.

¹⁸This paper obtains lower welfare costs than Veracierto [17] because it provides a quits allowance before taxing firing, while Veracierto [17] doesn't. As a consequence, the firing taxes have smaller effects.

the empirical plausibility of the business cycles generated by the benchmark economy with zero firing costs that was calibrated in Section 4. The first and second columns of Table 4 report business cycle statistics for the U.S. and the benchmark economy ($\tau = 0$), respectively. Before any statistics were computed, all time series were logged and detrended using the Hodrick-Prescott filter. The empirical measures of output, investment and consumption reported in the table correspond to the measures described in Section 4, and cover the period between 1960:1 and 1993:4. For the model economy, time series of length equal to 136 periods (the same length as the U.S. series) were computed for 100 simulations and the reported statistics are averages across these simulations. Comparing the business cycles generated by both economies, Table 4 shows that output fluctuates roughly the same amount in the model as in the U.S. Investment is about 5 times more variable than output in the model, while it is about 4 times as variable in the data. Consumption is less variable than output in both economies, however, it is less variable in the model than in the U.S. The aggregate stock of capital varies about the same amount in both economies. Hours vary less than output in the model, while they vary slightly more than output in the data. Similarly, productivity fluctuates less in the model than in the U.S. In terms of correlations with output, we see that almost all variables are highly procyclical, both in the model and in U.S. data. The only exceptions are capital (which is acyclical both in the model and the U.S.) and productivity (which is highly procyclical in the model, but acyclical in the data). Overall, the benchmark economy is found to be broadly consistent with salient features of U.S. business cycle dynamics, similarly to previous RBC models in the literature.

Having established the empirical plausibility of the benchmark economy, I now turn to evaluate the effects of firing costs on business cycle dynamics. The last three columns show the results. We see that introducing firing costs equal to one year of wages has considerable effects: The standard deviation of output decreases by 10.7% (from 1.40 to 1.25). The expenditures components also become less variable, but by different amounts: The standard deviation of consumption decreases by 4.1% (from 0.49 to 0.47) while the standard deviation of investment decreases by 14.7% (from 7.07 to 6.03). These are significant effects. However, they are relatively small compared to the effects on employment, whose standard deviation decreases by 30.6% (from 0.98 to 0.68). In terms of comovements with output, the effects

of firing taxes are generally insignificant. The only sizable effect is on labor productivity, whose correlation with output increases from 0.91 to 0.99.

The intuition for why firing taxes reduce the variability of aggregate employment by such a considerable amount is straightforward. The presence of firing taxes leads establishments to follow the (S,s) decision rule given by equation (3.29). As a consequence, an establishment that has a previous employment level net of quits $(1 - q)u$ between its lower employment threshold \underline{n} and its upper employment threshold \bar{n} chooses to remain inactive, that is, it makes its current employment equal to its previous employment level net of quits $(1 - q)u$. Since the employment level of an inactive establishment does not respond to aggregate conditions, the mere upsurge of this type of establishments leads to a reduction in the variability of aggregate employment.

The ranges of inaction also lowers the employment variability of active establishments. The reason is that an establishment that adjusts its employment level at a time of a positive aggregate shock will be concerned that, in the future, it may enter a long period of inaction during which aggregate productivity will revert to its mean. If the establishment responds too much to the current aggregate productivity shock it may find itself with an employment level that is too high for the aggregate productivity levels that will hold later on. For this reason, active establishments reduce the response of their employment levels \underline{n} and \bar{n} to the aggregate productivity shocks, dampening the fluctuations in aggregate employment.

Table 5 shows the effects of firing taxes on the persistence of aggregate employment. In particular, it reports the autocorrelation of aggregate employment at lags that vary between one and five quarters. We see that the autocorrelation at a one quarter lag increases from 0.66 to 0.71 when firing taxes equal to one year of wages ($\tau = 4w$) are introduced.

The effects are even larger at higher lags. For example, the autocorrelation at a three quarters lag increases from 0.20 to 0.27. These are significant effects. The reason for why firing taxes increase the persistence of aggregate employment can be found in the creation of the ranges of inaction. If an establishment enters a range of inaction after having expanded its employment level in response to an aggregate productivity shock, the effects of the expansion will persist until the establishment comes out of the range of inaction and adjusts its employment again. Thus the presence of inactive establishments makes aggregate employment more persistent.

Table 6 reports the effects of firing taxes on the cyclical behavior of job creation and job destruction. We see that when the firing taxes are equal to zero, the job creation rate and the job destruction rate vary by the same amount and are negatively correlated.¹⁹ When the firing taxes are introduced, the job creation rate and job destruction rates become less variable and less negatively correlated. An interesting feature in Table 6 is that the firing taxes decrease the variability of job destruction more than the variability of job creation. This finding seems to contradict Campbell and Fisher [4], who report that introducing firing costs increase the relative variability of job destruction. However, Campbell and Fisher perform a partial equilibrium analysis where the only source of fluctuations is a wage shock. Aggregate productivity, which is the source of aggregate fluctuations in this paper, is left unchanged. This is an important difference. Cabrales and Hopenhayn [3] find, in a partial equilibrium setting, that firing costs increase the relative variability of job creation when the source of the fluctuations is an aggregate productivity shock instead of the wage rate.

An interesting feature in Tables 4, 5 and 6 is that even low levels of firing taxes can have substantial effects on business cycle dynamics. In particular, firing taxes equal to one quarter of wages ($\tau = w$) are found to reduce the standard deviations of output, investment and labor 70% as much as firing taxes equal to one year of wages ($\tau = 4w$). The similarities are even stronger when considering the effects on the autocorrelation of aggregate employment, and the cyclical behavior job creation and job destruction. The reasons for why small firing taxes can have such significant effects are familiar to the literature. For example, in the investment decision problem analyzed by Abel and Eberly [1] the derivative of the range of inaction with respect to the wedge between the purchase and resale price of capital is shown to be infinite when the wedge is equal to zero. In this paper, the ranges of inaction are created by a firing tax but the mechanism is the same: Small firing taxes have large effects on the length of the ranges of inaction. Through their effects on the ranges of inaction, the small firing taxes have important consequences for the aggregate fluctuations of the economy.

¹⁹The fact that the job destruction rate is more variable than the job creation rate in U.S. data is not particularly worrisome. Veracierto [16] shows that incorporating a reallocation shock that is correlated with the aggregate productivity shock can reproduce that particular feature of U.S. data. However, the reallocation shock has no important effects on aggregate fluctuations.

Observe that, by reducing aggregate fluctuations, the welfare costs of firing taxes could be lower than those estimated from comparing steady states. However, this effect turns out to be negligible: Once the business cycle consequences are taken into account, the welfare costs of firing taxes are virtually the same as those reported in Table 3. There are two reasons for this. First, most of the reduction in variability takes place in employment. Since the preferences of the representative agent are linear with respect to this component, there are no welfare gains from this effect. Second, the volatility of consumption decreases but by a very small amount. This small effect, together with the relatively low risk aversion of the representative agent, produces an extremely small welfare gain.

5.3. No tax Rebates

So far, the firing taxes have been rebated to the representative household as a lump sum transfer. However, many of the firing costs paid by employers in actual countries do not go to the workers but involve resources that are wasted: Procedural requirements and legal costs are examples. To assess the effects of this type of firing costs, this section analyzes firing taxes that are thrown into the ocean. In particular, it analyzes a social planner problem given by maximizing equation (3.12) subject to equations (3.13) through (3.18), but where the lump-sum transfers $T_t(z^t)$ in equation (3.13) are set to zero.

Table 7 reports the steady state results. Not surprisingly, the effects of firing taxes on the job creation and destruction process are the same as when the firing taxes are rebated as lump sum transfers. However, there are important differences in the rest of the variables. In particular, firing taxes equal to one year of wages ($\tau = 4w$) reduce employment by only 0.28%, compared to the 2.46% drop reported in Table 3. The reason is that when the firing taxes are not rebated to the household sector, they generate a large income effect. This effect cancels the substitution effect from the lower wages and leads to a small change in labor supply. Despite the small drop in employment, consumption decreases quite considerably because part of the output is used up in firing workers. Since both consumption and leisure are smaller than when the firing taxes are rebated as lump-sum transfers, the welfare costs are much larger: 3.84% instead of 1.74%.

Table 8 reports the business cycle effects. We see that the business cycle fluctuations are

virtually the same when the tax revenues are rebated to the households sector as when they are not (Table 4). There are two reasons for this. First, total tax revenues are small: Even when τ is equal to one year of wages and the tax revenues are the largest, they represent only 1.86% of aggregate output (Tables 3 and 7). Second, the tax revenues fluctuate very little: When τ is equal to one year of wages, their standard deviation is only 0.50. Since the tax revenues are small and fluctuate very little, it makes no difference if they are rebated to the households or not: They do not represent a significant source of aggregate fluctuations.

We conclude that determining to what extent the firing taxes are rebated to the household sector is crucial for evaluating their long-run outcomes, but has no importance for analyzing their business cycle effects.

A. Equivalence between quasi-planner and competitive equilibria

Let

$$\beta^t \lambda_t(z^t) \left[\prod_{j=1}^t H(z_{j-1}, z_j) \right],$$

$$\left[\prod_{h=1}^j p_{t+h-1}(z^{t+h-1}, z_{t+h}) \right] \left[\prod_{h=1}^j Q(s_{a+h-1}, s_{a+h}) \right] \phi_{a+j,t+j}(s^{a+j}, z^{t+j}),$$

$$\left[\prod_{h=1}^j p_{t+h-1}(z^{t+h-1}, z_{t+h}) \right] \left[\prod_{h=1}^j Q(s_{a+h-1}, s_{a+h}) \right] \xi_{a+j,t+j}(s^{a+j}, z^{t+j})$$

be the Lagrange multipliers for equations (3.2)-(3.4), respectively. The first order conditions for a competitive equilibrium are then given by:

$$\frac{1}{c_t(z^t)} = \lambda_t(z^t) \tag{A.1}$$

$$\alpha = \lambda_t(z^t) w_t(z^t) \tag{A.2}$$

$$\lambda_t(z^t) = \beta \sum_{z_{t+1}} \lambda_{t+1}(z^{t+1}) H(z_t, z_{t+1}) [r_{t+1}(z^{t+1}) + 1 - \delta] \tag{A.3}$$

$$\lambda_t(z^t) p_t(z^t; z_{t+1}) = \beta \lambda_{t+1}(z^{t+1}) H(z_t, z_{t+1}) \tag{A.4}$$

$$e^{z_t} s_a g_{a,t}(s^a, z^t)^\theta \gamma n_{a,t}(s^a, z^t)^{\gamma-1} - w_t(z^t) + \phi_{a,t}(s^a, z^t) -$$

$$\sum_{s_{a+1}} \sum_{z_{t+1}} (1-q) \phi_{a+1,t+1}(s^{a+1}, z^{t+1}) p_t(z^{t+1}) Q(s_a, s_{a+1}) \leq 0, (=0, \text{ if } n_{a,t}(s^a, z^t) > 0) \quad (\text{A.5})$$

$$e^{z^t} s_a \theta g_{a,t}(s^a, z^t)^{\theta-1} n_{a,t}(s^a, z^t)^\gamma - r_t(z^t) \leq 0, (=0, \text{ if } g_{a,t}(s^a, z^t) > 0) \quad (\text{A.6})$$

$$-\tau + \phi_{a,t}(s^a, z^t) + \xi_{a,t}(s^a, z^t) \leq 0, (=0, \text{ if } f_{a,t}(s^a, z^t) > 0) \quad (\text{A.7})$$

$$\phi_{a,t}(s^a, z^t) [n_{a,t}(s^a, z^t) - (1-q) n_{a-1,t-1}(s^{a-1}, z^{t-1}) + f_{a,t}(s^a, z^t)] = 0 \quad (\text{A.8})$$

$$\xi_{a,t}(s^a, z^t) f_{a,t}(s^a, z^t) = 0, \quad (\text{A.9})$$

and equations (3.2)-(3.11).

Let

$$\begin{aligned} & \beta^t \lambda_t(z^t) \left[\prod_{j=1}^t H(z_{j-1}, z_j) \right], \\ & \beta^t \lambda_t(z^t) \left[\prod_{j=1}^t H(z_{j-1}, z_j) \right] \phi_{a,t}(s^a, z^t) \mu_a(s^a), \\ & \beta^t \lambda_t(z^t) \left[\prod_{j=1}^t H(z_{j-1}, z_j) \right] w_t(z^t), \\ & \beta^t \lambda_t(z^t) \left[\prod_{j=1}^t H(z_{j-1}, z_j) \right] r_t(z^t), \\ & \beta^t \lambda_t(z^t) \left[\prod_{j=1}^t H(z_{j-1}, z_j) \right] \xi_{a,t}(s^a, z^t) \mu_a(s^a) \end{aligned}$$

be the Lagrange multipliers for equations (3.13)-(3.17), respectively. The first order conditions for a quasi-planner equilibrium are then given by equations (A.1)-(A.9) and equations (3.13)-(3.19).

Establishing that quasi-planner and competitive equilibria are equivalent then amounts to showing that equations (3.2)-(3.11) are satisfied if and only if equations (3.13)-(3.19) hold. This is a straightforward verification.

B. Determination of the individual state of an establishment

Proposition B.1. *Let (h, s^h) be the age and the idiosyncratic history of a particular type of establishment at date 0. Let (j, s^j) be the age and the idiosyncratic history of another type of establishment at date 0. Suppose that*

$$n_{h-1,-1}(s^{h-1}, z^{-1}) = n_{j-1,-1}(s^{j-1}, z^{-1}),$$

and that $s_h = s_j$.

Then, the solution $\{\hat{c}_t, \hat{k}_{t+1}, \hat{\eta}_t, \hat{g}_t, \hat{n}_t, \hat{f}_t\}_{t=0}^{\infty}$ to the quasi-planner problem (3.12) has the following property:

$$\begin{aligned}\hat{n}_{h,0}(s^h, z^0) &= \hat{n}_{j,0}(s^j, z^0), \\ \hat{f}_{h,0}(s^h, z^0) &= \hat{f}_{j,0}(s^j, z^0) \\ \hat{g}_{h,0}(s^h, z^0) &= \hat{g}_{j,0}(s^j, z^0),\end{aligned}$$

and for every $t > 1$, s^t and z^t .

$$\begin{aligned}\hat{n}_{h+t}((s^h, s^t), z^t) &= \hat{n}_{j+t}((s^j, s^t), z^t), \\ \hat{f}_{h+t}((s^h, s^t), z^t) &= \hat{f}_{j+t}((s^j, s^t), z^t). \\ \hat{g}_{h+t}((s^h, s^t), z^t) &= \hat{g}_{j+t}((s^j, s^t), z^t),\end{aligned}$$

Proof: Suppose not.

Let

$$\begin{aligned}n_0^\psi(z^0) &= \psi_0 \hat{n}_{h,0}(s^h, z^0) + [1 - \psi_0] \hat{n}_{j,0}(s^j, z^0), \\ f_0^\psi(z^0) &= \psi_0 \hat{f}_{h,0}(s^h, z^0) + [1 - \psi_0] \hat{f}_{j,0}(s^j, z^0), \\ g_0^\psi(z^0) &= \psi_0 \hat{g}_{h,0}(s^h, z^0) + [1 - \psi_0] \hat{g}_{j,0}(s^j, z^0),\end{aligned}$$

where

$$\psi_0 = \frac{\mu_h(s^h)}{\mu_h(s^h) + \mu_j(s^j)},$$

and, for every $t > 1$, s^t and z^t , let

$$n_t^\psi (s^t, z^t) = \psi_t (s^t) \hat{n}_{h+t} ((s^h, s^t), z^t) + [1 - \psi_t (s^t)] \hat{n}_{j+t} ((s^j, s^t), z^t),$$

$$f_t^\psi (s^t, z^t) = \psi_t (s^t) \hat{f}_{h+t} ((s^h, s^t), z^t) + [1 - \psi_t (s^t)] \hat{f}_{j+t} ((s^j, s^t), z^t),$$

$$g_t^\psi (s^t, z^t) = \psi_t (s^t) \hat{g}_{h+t} ((s^h, s^t), z^t) + [1 - \psi_t (s^t)] \hat{g}_{j+t} ((s^j, s^t), z^t),$$

where

$$\psi_t (s^t) = \frac{\mu_{h+t} ((s^h, s^t))}{\mu_{h+t} ((s^h, s^t)) + \mu_{j+t} ((s^j, s^t))}.$$

Consider an alternative contingent plan $\{\tilde{k}_{t+1}, \tilde{\eta}_t, \tilde{g}_t, \tilde{n}_t, \tilde{f}_t\}_{t=0}^\infty$ which is identical to the solution to the quasi-planner problem except that

$$\tilde{n}_{h,0} (s^h, z^0) = \tilde{n}_{j,0} (s^j, z^0) = n_0^\psi (z^0)$$

$$\tilde{f}_{h,0} (s^h, z^0) = \tilde{f}_{j,0} (s^j, z^0) = f_0^\psi (z^0)$$

$$\tilde{g}_{h,0} (s^h, z^0) = \tilde{g}_{j,0} (s^j, z^0) = g_0^\psi (z^0)$$

and for every $t > 1$, s^t and z^t :

$$\tilde{n}_{h+t} ((s^h, s^t), z^t) = \tilde{n}_{j+t} ((s^j, s^t), z^t) = n_t^\psi (s^t, z^t)$$

$$\tilde{f}_{h+t} ((s^h, s^t), z^t) = \tilde{f}_{j+t} ((s^j, s^t), z^t) = f_t^\psi (s^t, z^t).$$

$$\tilde{g}_{h+t} ((s^h, s^t), z^t) = \tilde{g}_{j+t} ((s^j, s^t), z^t) = g_t^\psi (s^t, z^t)$$

This alternative plan is feasible and, by the strict concavity of the establishment level production function, it leads to a larger right hand side to equation (3.13) for every z^t . Hence, consumption can be made larger than under the optimal plan (strictly larger at some z^t) while aggregate employment is left unchanged. This increases expected utility, leading to a contradiction.

C. Characterization of the optimal employment rule

From equations (3.14), (A.7)-(A.9) we know that

$$0 \leq \phi_{a,t}(s^a, z^t) \leq \tau \quad (\text{C.1})$$

$$n_{a,t}(s^a, z^t) > (1-q)n_{a-1,t-1}(s^{a-1}, z^{t-1}) \implies \phi_{a,t}(s^a, z^t) = 0 \quad (\text{C.2})$$

$$n_{a,t}(s^a, z^t) < (1-q)n_{a-1,t-1}(s^{a-1}, z^{t-1}) \implies \phi_{a,t}(s^a, z^t) = \tau \quad (\text{C.3})$$

$$0 < \phi_{a,t}(s^a, z^t) < \tau \implies n_{a,t}(s^a, z^t) = (1-q)n_{a-1,t-1}(s^{a-1}, z^{t-1}) \quad (\text{C.4})$$

Using equations (A.5), (A.6), and (C.1)-(C.4) we have (when $s_a > 0$) that:

$$\phi_{a,t}(s^a, z^t) = \min \{ \tau, \max [\Omega_{a,t}(s^a, z^t), 0] \} \quad (\text{C.5})$$

where

$$\begin{aligned} \Omega_{a,t}(s^a, z^t) &= w_t(z^t) - (e^{z^t} s_a)^{\frac{1}{1-\theta}} \left(\frac{\theta}{r_t(z^t)} \right)^{\frac{\theta}{1-\theta}} \gamma [(1-q)n_{a-1,t-1}(s^{a-1}, z^{t-1})]^{-\left(\frac{1-\theta-\gamma}{1-\theta}\right)} \\ &+ \sum_{s_{a+1}} \sum_{z^{t+1}} (1-q) \phi_{a+1,t+1}(s^{a+1}, z^{t+1}) p_t(z^{t+1}) Q(s_a, s_{a+1}). \end{aligned}$$

Under a recursive formulation, $\phi_{a,t}(s^a, z^t)$, $\Omega_{a,t}(s^a, z^t)$ and $n_{a,t}(s^a, z^t)$ will depend on $(u, s; z, K, X, k, x)$, where u is the previous period employment $n_{a-1,t-1}(s^{a-1}, z^{t-1})$ and s is the current idiosyncratic productivity level s_a , while $w_t(z^t)$, $r_t(z^t)$, and $p_t(z^{t+1})$ will depend on (z, K, X, k, x) . Abusing notation, for $s > 0$, we can rewrite equation (C.5) as follows:

$$\phi(u, s; z, K, X, k, x) = \min \{ \tau, \max [\Omega(u, s; z, K, X, k, x), 0] \} \quad (\text{C.6})$$

where

$$\Omega(u, s; z, K, X, k, x) = w(z, K, X, k, x) - (e^z s)^{\frac{1}{1-\theta}} \left(\frac{\theta}{r(z, K, X, k, x)} \right)^{\frac{\theta}{1-\theta}} \gamma [(1-q)u]^{-\left(\frac{1-\theta-\gamma}{1-\theta}\right)}$$

$$+ \sum_{s'} \sum_{z'} (1-q) \phi((1-q)u, s'; z', K', X', k', x') p(z, K, X, k, x; z') Q(s, s')$$

and where K' , X' , k' and x' are given by equations (3.26), (3.27), (3.23) and (3.24) respectively.

Also,

$$\phi(u, 0; z, K, X, k, x) = \tau \quad (\text{C.7})$$

since establishments that receive an idiosyncratic productivity s equal to zero choose zero employment levels.²⁰

Note that, since

$$- (e^z s)^{\frac{1}{1-\theta}} \left(\frac{\theta}{r(z, K, X, k, x)} \right)^{\frac{\theta}{1-\theta}} \gamma [(1-q)u]^{-\left(\frac{1-\theta-\gamma}{1-\theta}\right)}$$

is strictly increasing in u , the solution ϕ to the functional equation given by (C.6) and (C.7) will be increasing in u . As a consequence, there exists a unique $\bar{n}(s; z, K, X, k, x)$ satisfying

$$\begin{aligned} \tau &= w(z, K, X, k, x) - (e^z s)^{\frac{1}{1-\theta}} \left(\frac{\theta}{r(z, K, X, k, x)} \right)^{\frac{\theta}{1-\theta}} \gamma [\bar{n}(s; z, K, X, k, x)]^{-\left(\frac{1-\theta-\gamma}{1-\theta}\right)} \\ &+ \sum_{s'} \sum_{z'} (1-q) \phi(\bar{n}(s; z, K, X, k, x), s'; z', K', X', k', x') p(z, K, X, k, x; z') Q(s, s'), \end{aligned} \quad (\text{C.8})$$

and there exists a unique $\underline{n}(s; z, K, X, k, x)$ satisfying

$$\begin{aligned} 0 &= w(z, K, X, k, x) - (e^z s)^{\frac{1}{1-\theta}} \left(\frac{\theta}{r(z, K, X, k, x)} \right)^{\frac{\theta}{1-\theta}} \gamma [\underline{n}(s; z, K, X, k, x)]^{-\left(\frac{1-\theta-\gamma}{1-\theta}\right)} \\ &+ \sum_{s'} \sum_{z'} (1-q) \phi(\underline{n}(s; z, K, X, k, x), s'; z', K', X', k', x') p(z, K, X, k, x; z') Q(s, s'). \end{aligned} \quad (\text{C.9})$$

Observe that $\underline{n}(s; z, K, X, k, x) < \bar{n}(s; z, K, X, k, x)$.

Since

$$n(u, s; z, K, X, k, x) > (1-q)u \implies \phi(u, s; z, K, X, k, x) = 0,$$

²⁰This is true if τ is sufficiently small relative to the present discounted value of wages

$$n(u, s; z, K, X, k, x) < (1 - q)u \implies \phi(u, s; z, K, X, k, x) = \tau,$$

$$0 < \phi(u, s; z, K, X, k, x) < \tau \implies n(u, s; z, K, X, k, x) = (1 - q)u,$$

and since

$$\begin{aligned} \phi(u, s; z, K, X, k, x) &= w(z, K, X, k, x) \\ &- (e^z s)^{\frac{1}{1-\theta}} \left(\frac{\theta}{r(z, K, X, k, x)} \right)^{\frac{\theta}{1-\theta}} \gamma [n(u, s; z, K, X, k, x)]^{-\left(\frac{1-\theta-\gamma}{1-\theta}\right)} \\ &+ \sum_{s'} \sum_{z'} (1 - q) \phi(n(u, s; z, K, X, k, x), s'; z', K', X', k', x') p(z, K, X, k, x; z') Q(s, s'), \end{aligned}$$

equations (C.8) and (C.9), together with the fact that ϕ is increasing in u , imply the employment decision rule (3.29).

D. Support of invariant distribution

Proof of Proposition 3.1: That 0 belongs to the support follows from the fact that new establishments are born with zero previous period employment and from the fact that establishments that die (i.e transit to $s = 0$) choose zero employment level.

That the set m^* belongs to the support follows from the fact that every time that an establishment of type (u, s) has a next period number of agents different from $n' = (1 - q)u$ it must be $n' = (1 - q)\underline{n}^*(s)$, if the establishment expands, or $n' = (1 - q)\bar{n}^*(s)$, if the establishment contracts. Observe that $\underline{\Omega}(s)$ is an upper-bound on the duration of inaction for an establishment that has just expanded (and has current idiosyncratic productivity $s \geq s_{\min}$). Similarly, $\bar{\Omega}(s)$ is an upper-bound on the duration of inaction for an establishment that has just contracted (and has current idiosyncratic productivity $s \geq s_{\min}$).

FIGURE 1
Output fluctuations vs. employment protection

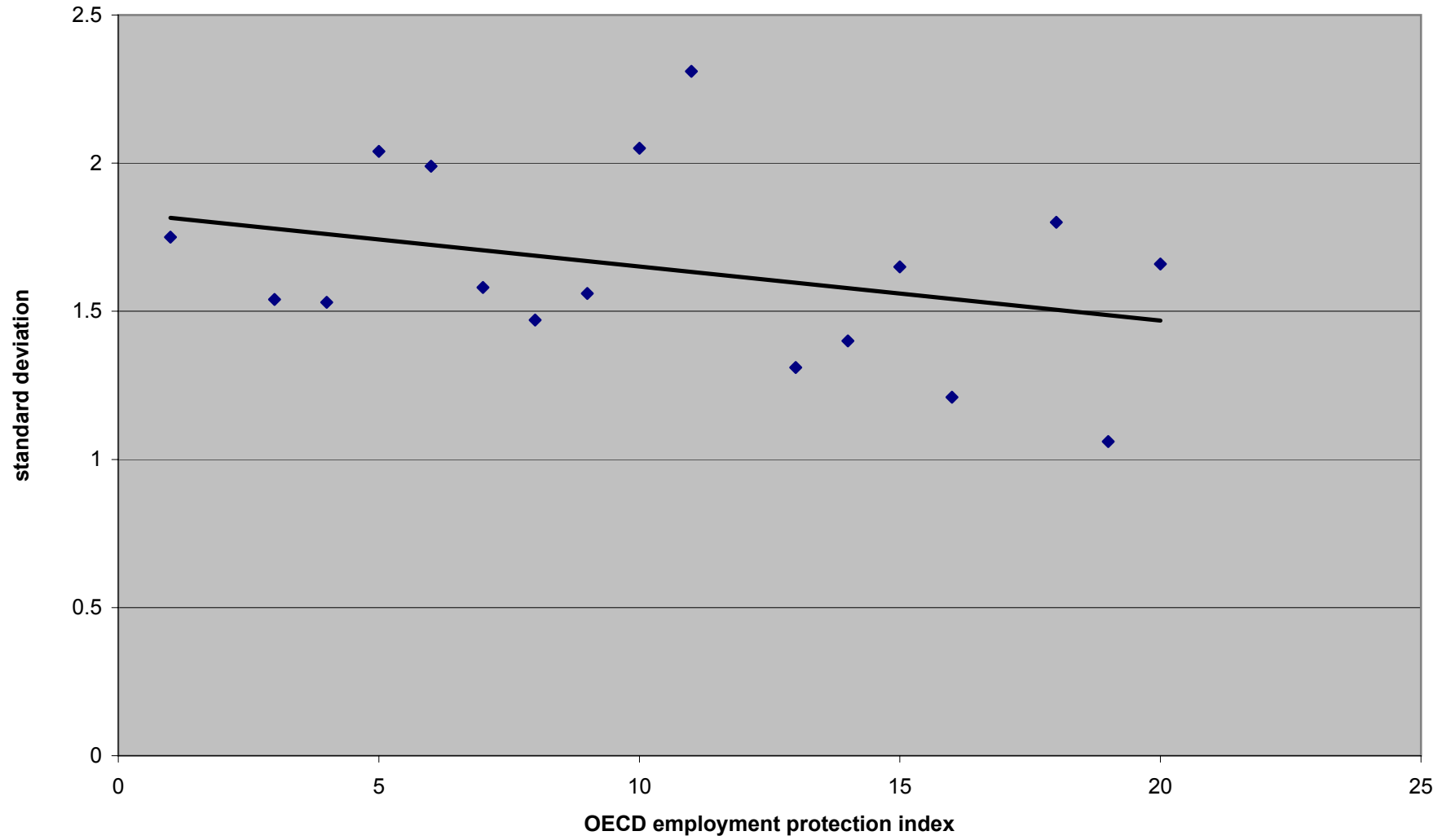


Table 1

Size distribution of U.S. manufacturing establishments

Employment	Shares (%)
5-9	23.15
10-19	22.82
20-49	24.83
50-99	12.59
100-249	10.05
250-499	3.86
500-999	1.68
1000-2499	0.73
>2500	0.28

Table 2

Calibrated idiosyncratic process

Idiosyncratic Productivity levels:

$$\begin{array}{cccccc}
 s_0 = 0.00 & s_1 = 1.00 & s_2 = 1.11 & s_3 = 1.26 & s_4 = 1.40 & \\
 s_5 = 1.58 & s_6 = 1.76 & s_7 = 1.94 & s_8 = 2.18 & s_9 = 2.53 &
 \end{array}$$

Initial distribution:

$$\begin{array}{cccccc}
 \psi_0 = 0.00 & \psi_1 = 0.50 & \psi_2 = 0.15 & \psi_3 = 0.35 & \psi_4 = 0.00 & \\
 \psi_5 = 0.00 & \psi_6 = 0.00 & \psi_7 = 0.00 & \psi_8 = 0.00 & \psi_9 = 0.00 &
 \end{array}$$

Transition matrix:

$$Q = \begin{pmatrix}
 1.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
 0.088 & 0.847 & 0.065 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
 0.005 & 0.084 & 0.879 & 0.032 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
 0.005 & 0.000 & 0.086 & 0.847 & 0.062 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 \\
 0.005 & 0.000 & 0.000 & 0.088 & 0.876 & 0.031 & 0.000 & 0.000 & 0.000 & 0.000 \\
 0.005 & 0.000 & 0.000 & 0.000 & 0.090 & 0.846 & 0.059 & 0.000 & 0.000 & 0.000 \\
 0.005 & 0.000 & 0.000 & 0.000 & 0.000 & 0.092 & 0.808 & 0.095 & 0.000 & 0.000 \\
 0.005 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.094 & 0.873 & 0.028 & 0.000 \\
 0.005 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.096 & 0.895 & 0.004 \\
 0.005 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.000 & 0.099 & 0.896
 \end{pmatrix}$$

Table 3

Steady state effects

	$\tau = 0$	$\tau = 0.33w$	$\tau = w$	$\tau = 2w$	$\tau = 4w$
output	100.00	99.13	98.38	97.65	96.48
consumption	100.00	99.13	98.38	97.65	96.48
investment	100.00	99.13	98.38	97.65	96.48
capital	100.00	99.13	98.38	97.65	96.48
labor	100.00	99.31	98.99	98.55	97.54
taxes/output	0.0%	0.49%	0.78%	1.13%	1.86%
JCB	0.73%	0.66%	0.58%	0.53%	0.47%
JCC	4.80%	3.71%	3.17%	3.05%	3.05%
JDD	0.73%	0.77%	0.79%	0.80%	0.80%
JDC	4.80%	3.61%	2.96%	2.79%	2.74%
Welfare cost	0.00%	0.35%	0.88%	1.30%	1.74%

Table 4

Business cycle effects

	U.S. data	$\tau = 0$	$\tau = 0.33w$	$\tau = w$	$\tau = 2w$	$\tau = 4w$
A: Standard deviations						
output	1.33	1.40	1.35	1.29	1.26	1.25
consumption	0.87	0.49	0.49	0.48	0.47	0.47
investment	4.99	7.07	6.72	6.32	6.12	6.03
capital	0.63	0.50	0.49	0.47	0.46	0.45
labor	1.42	0.98	0.88	0.77	0.71	0.68
productivity	0.76	0.49	0.52	0.55	0.57	0.58
taxes	n.a.	n.a.	2.27	1.27	0.71	0.50
B: Correlations with output						
output	1.00	1.00	1.00	1.00	1.00	1.00
consumption	0.91	0.91	0.92	0.92	0.93	0.93
investment	0.91	0.98	0.98	0.98	0.98	0.98
capital	0.04	0.08	0.08	0.08	0.08	0.08
labor	0.85	0.98	0.98	0.99	0.99	0.99
productivity	-0.16	0.91	0.95	0.97	0.98	0.99
taxes	n.a.	n.a.	0.26	0.20	0.29	0.54

Table 5

Employment autocorrelation function

	$\tau = 0$	$\tau = 0.33w$	$\tau = w$	$\tau = 2w$	$\tau = 4w$
1 quarter	0.66	0.69	0.71	0.71	0.71
2 quarters	0.40	0.44	0.46	0.47	0.47
3 quarters	0.20	0.24	0.26	0.27	0.27
4 quarters	0.03	0.06	0.08	0.09	0.09
5 quarters	-0.09	-0.07	-0.05	-0.04	-0.04

Table 6

Job creation and job destruction fluctuations

	U.S.	$\tau = 0$	$\tau = 0.33w$	$\tau = w$	$\tau = 2w$	$\tau 4w$
$\sigma(\text{JC})$	0.88	0.44	0.39	0.34	0.32	0.31
$\sigma(\text{JD})$	1.65	0.44	0.37	0.30	0.28	0.27
$\text{corr}(\text{JC}, \text{JD})$	-0.37	-0.60	-0.60	-0.57	-0.57	-0.56

Table 7

Steady state effects - No tax rebates

	$\tau = 0$	$\tau = 0.33w$	$\tau = w$	$\tau = 2w$	$\tau = 4w$
output	100.00	99.60	99.12	98.72	98.25
consumption	100.00	99.03	98.22	97.41	96.10
investment	100.00	99.60	99.12	98.72	98.25
capital	100.00	99.60	99.12	98.72	98.25
labor	100.00	99.88	99.90	99.88	99.72
taxes/output	0.0%	0.49%	0.78%	1.13%	1.86%
JCB	0.73%	0.66%	0.58%	0.53%	0.47%
JCC	4.80%	3.71%	3.17%	3.05%	3.05%
JDD	0.73%	0.77%	0.79%	0.80%	0.80%
JDC	4.80%	3.61%	2.96%	2.79%	2.74%
Welfare cost	0.00%	0.89%	1.74%	2.56%	3.84%

Table 8

Business cycle effects - No tax rebates

	U.S. data	$\tau = 0$	$\tau = 0.33w$	$\tau = w$	$\tau = 2w$	$\tau = 4w$
A: Standard deviations						
output	1.33	1.40	1.35	1.29	1.26	1.24
consumption	0.87	0.49	0.49	0.48	0.47	0.47
investment	4.99	7.07	6.74	6.32	6.14	6.02
capital	0.63	0.50	0.49	0.47	0.46	0.45
labor	1.42	0.98	0.88	0.77	0.71	0.68
productivity	0.76	0.49	0.52	0.55	0.57	0.58
taxes	n.a.	n.a.	2.28	1.27	0.72	0.50
B: Correlations with output						
output	1.00	1.00	1.00	1.00	1.00	1.00
consumption	0.91	0.91	0.92	0.92	0.93	0.93
investment	0.91	0.98	0.98	0.98	0.98	0.98
capital	0.04	0.08	0.08	0.08	0.09	0.08
labor	0.85	0.98	0.98	0.99	0.99	0.99
productivity	-0.16	0.91	0.95	0.97	0.98	0.99
taxes	n.a.	n.a.	0.26	0.20	0.28	0.54

References

- [1] Abel, Andrew B. and Eberly, Janice C. “Optimal Investment with Costly Reversibility.” *Review of Economic Studies*, October 1996, 63(4), pp. 581–93.
- [2] Alvarez, F. and Veracierto, M., 2001, “Search, Self-Insurance and Job Security Provisions”, *Journal of Monetary Economics*, v47, pp. 477-498.
- [3] Cabrales, A. and H. Hopenhayn, 1997, “Labor-market flexibility and aggregate employment volatility”, *Carnegie-Rochester Conference Series on Public Policy*, 46, 189-228.
- [4] Campbell, J. and J. Fisher, 2000, “Aggregate Employment Fluctuations with Microeconomic Asymmetries”, *American Economic Review*, 90, 1323-45.
- [5] Davis, S. and J. Haltiwanger. 1990. Gross Job Creation and Destruction: Microeconomic Evidence and Macroeconomic Implications. *NBER Macroeconomics Annual*, 5, 123-168.
- [6] Dunne, T., M. Roberts and L. Samuelson. 1989. The Growth and Failure of U.S. Manufacturing Plants. *Quarterly Journal of Economics*, 104, 671-698.
- [7] Hansen, G. 1985. Indivisible Labor and the Business Cycle. *Journal of Monetary Economics*, 16, 309-327.
- [8] Hansen, G. and Edward C. Prescott. 1995. Recursive Methods for Computing Equilibria of Business Cycle Models. In T. F. Cooley, ed., *Frontiers of Business Cycle Research*. Princeton: Princeton University Press.
- [9] Hopenhayn, H. and Rogerson, Richard. 1993. Job Turnover and Policy Evaluation: A General Equilibrium Analysis. *Journal of Political Economy*, 101:5, 915-38.
- [10] Lazear, E. 1990. Job Security Provisions and Employment. *Quarterly Journal of Economics*, 105, 699-726.
- [11] Mehra, R. and E. Prescott. 1985. The Equity Premium: A Puzzle. *Journal of Monetary Economics*, 15, 145-161.

- [12] Millard, S. and D. Mortensen, 1997, “The Unemployment and Welfare Effects of Labour Market Policy: A Comparison of the U.S. and the U.K.”, in *Unemployment Policy: How Should Governments Respond to Unemployment*, edited by Dennis Snower and Guillermo de la Dehesa, Oxford University Press.
- [13] Prescott, Edward C. 1986. Theory Ahead of Business Cycle Measurement. *Federal Reserve Bank of Minneapolis Quarterly Review*, 10:4, 9-22.
- [14] Rogerson, R. 1988. Indivisible Labor, Lotteries and Equilibrium. *Journal of Monetary Economics*, 21:1, 3-16.
- [15] Samaniego, R. 2003. Employment Protection and Macroeconomic Dynamics. George Washington University, mimeo.
- [16] Veracierto, M. 1995. Essays on Job Creation and Job Destruction. University of Minnesota, Ph.D. dissertation.
- [17] Veracierto, M. 2001. Employment Flows, Capital Mobility, and Policy Analysis. *International Economic Review*, 42:3, 571-595.